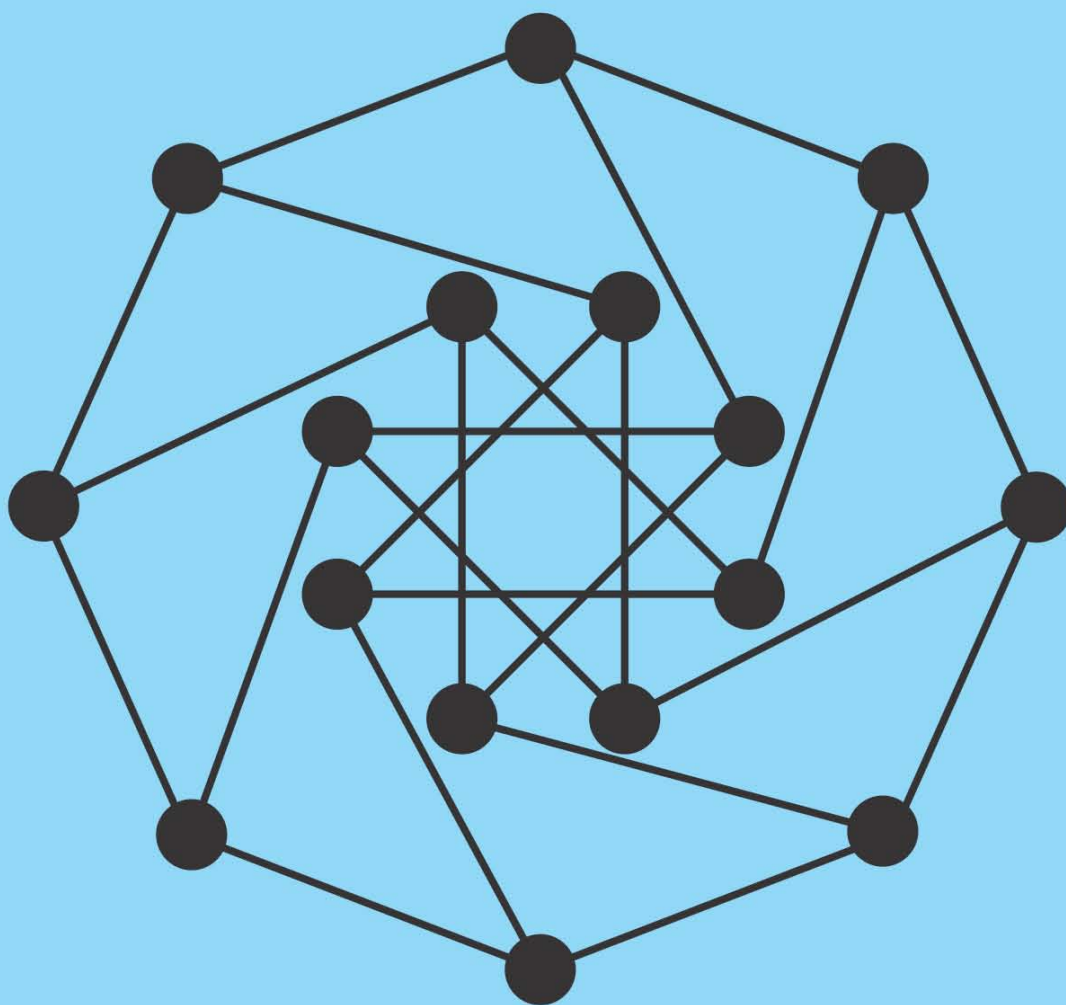


# Introduction to Graph Theory

*drawing*

A. P. Santika



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# 1. Introduction

## 1.1 Components

Components included in this game are:

- 30 resource cards
- 60 objective cards (30 S objectives and 30 C objectives)
- 18 Bonus cards
- 18 Extra Power cards
- Player sheets

## 1.2 Overview of the Game

In this game, we will draw simple graphs using resources available to us: vertices, edges, and Eulerian (this resource acts as currency). To draw a graph, we need vertices and edges. We use Eulerian to use the effect of an active Extra Power.

Each round we will choose one resource card from the ones available in that round, one resource card usually contains a number of two different resources. There is also an active Extra Power for that round. This active Extra Power will change every round. For more details on how the game play, read **Chapter 2**.

There are also Objective cards available for us to claim if the graph we drew satisfies the requirement for them. There are two kinds of Objective cards, S (for subgraph) Objective and C (for component) Objective. We will go into details on the difference between these types in **Subsection 1.4.3**.

## 1.3 Drawing Simple Graphs

A simple graph consists of vertices (nodes or dots) and edges (lines) with the following restrictions:

1. Each edge connects **exactly** two **different** vertices.
2. There is **at most** one edge connecting two vertices.

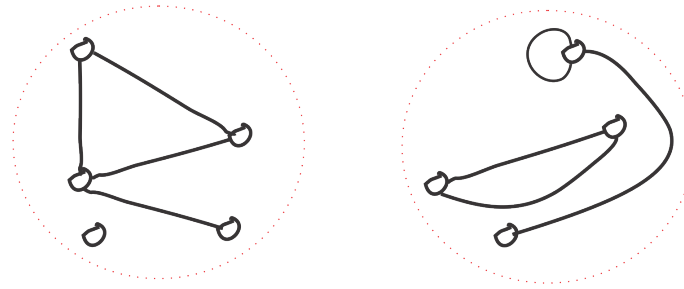


Figure 1.1: Examples of graphs

■ **Example 1.1** The example on the left is a simple graph because every edge connects two different vertices and every two vertices are only connected by one edge. Meanwhile, the example on the right contains one edge connecting the same vertex and two vertices connected by two different edges. So, this is a non-simple graph. ■

A few things to note when we draw edges are:

- we can only draw edges if there are two vertices that are still unconnected
- edges can be straight or curved
- the intersection of two or more edges is not a vertex

Please also note that we cannot place a vertex in the intersection of two or more edges. This is done to avoid confusion.

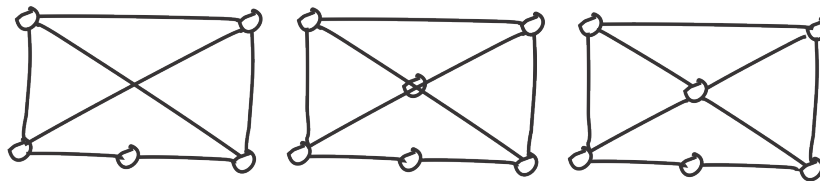


Figure 1.2: Intersection example

■ **Example 1.2** The graph on the left has two edges intersecting, the graph in the middle has two edges intersecting but also one vertex drawn right in the intersection. The only difference between these two graphs is the vertex in the intersection. This vertex does not change any configuration of the graph on the left.

On the other hand, the graph on the right contains no intersection between edges. Furthermore, this graph's edge configuration is totally different from the other two. While the other two graphs have 7 edges, this graph has 9 edges. ■

## 1.4 Anatomy of Player Sheet and Cards

In this section, we will take a look at player sheet and every different kind of cards. There are five different kinds of cards and 126 cards in total.

### 1.4.1 Player Sheet

This is the sheet that we will be using to play in this game and short explanation of each part of the sheet.

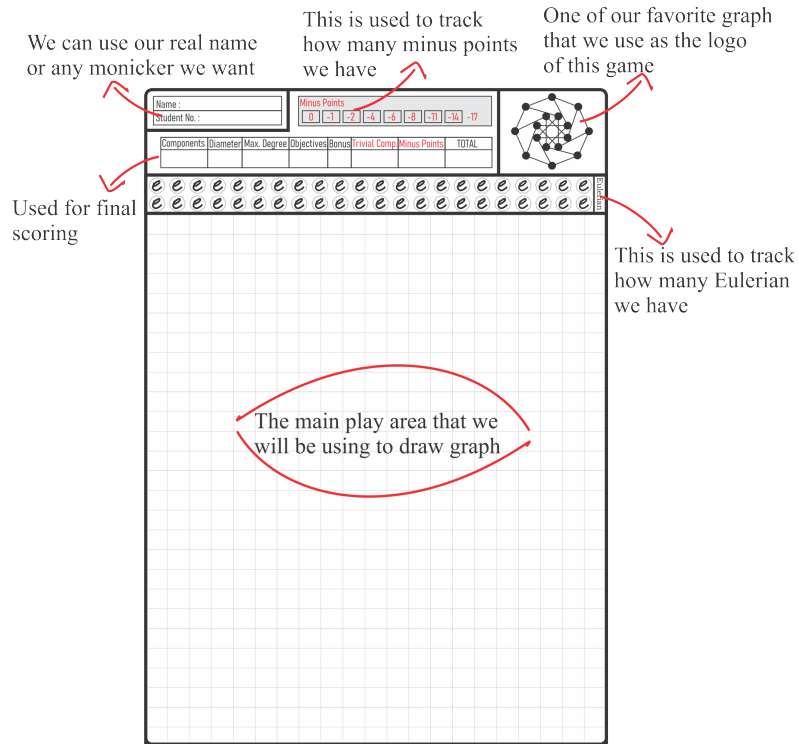


Figure 1.3: Examples of Resource card

The Eulerian track is used this way:

- Whenever we gain  $N$  Eulerians, circle  $N$  number of Eulerian coins in the track.
- Whenever we spent  $X$  Eulerians, cross  $X$  number of already circled Eulerian coins in the track. We cannot spend uncircled Eulerian coins or a crossed and circled Eulerian coins.

### 1.4.2 Resource Cards

When chosen, these cards will give us resources printed in the chosen card. We use these symbols for convenience:  $\bullet$  for vertices,  $\backslash$  for edges, and  $e$  for Eulerians.

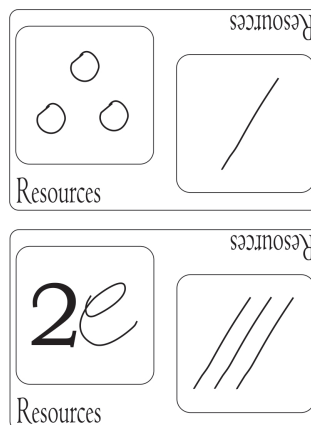


Figure 1.4: Examples of Resource card

If we chose the card on the top, we must draw 3 vertices and 1 edges, while the card on the bottom will give us 2 Eulerians and we also have to draw 3 edges.

### 1.4.3 Objective Cards

There are two types of objectives, S (for subgraph) and C (for component).

- S Objectives: player can modify the graph after the objective is claimed.
- C Objectives: player cannot modify the graph after the objective is claimed (after the objective is claimed, **circle all the components needed for the objective**, these components cannot be modified).

The term "modify" means adding edges or connecting other vertices to that particular graph.

- R** Even though we cannot modify any graph we claim for C Objective card, it is still possible to use this graph to claim other Objective cards.

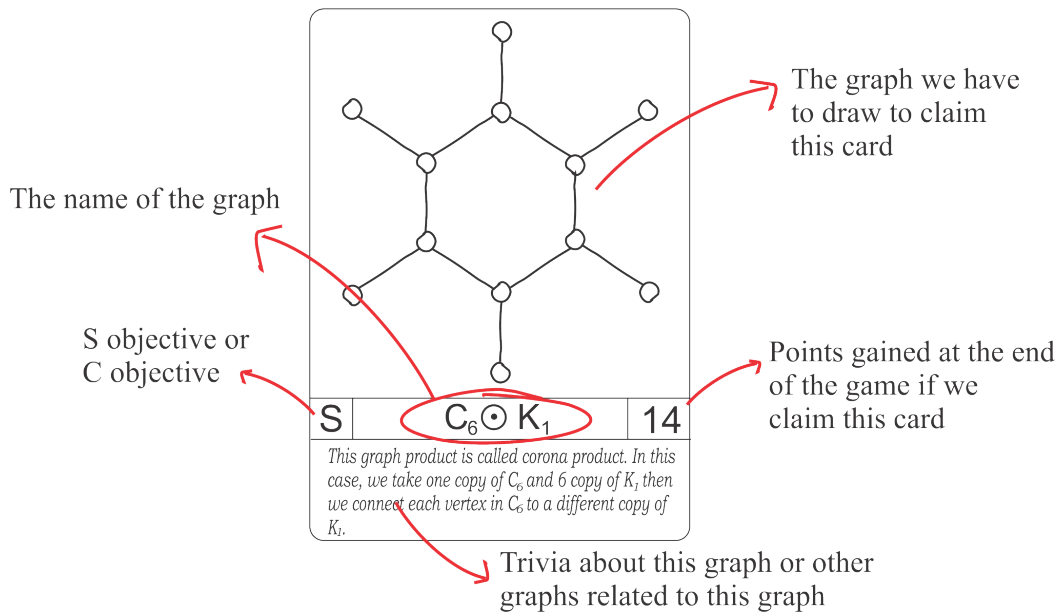


Figure 1.5: Objective card anatomy

It is on the player's responsibility to notice that they have fulfilled the requirement for a certain Objective card and if other players ask them to prove it, then the player must do so.

Name : _____	<b>Minus Points</b> <input checked="" type="checkbox"/> <input checked="" type="checkbox"/> <input checked="" type="checkbox"/> <input checked="" type="checkbox"/> <input type="checkbox"/> -6   <input type="checkbox"/> -8   <input type="checkbox"/> -11   <input type="checkbox"/> -14   <input type="checkbox"/> -17						
Student No. : _____							
Components	Diameter	Max. Degree	Objectives	Bonus	Trivial Comp.	Minus Points	TOTAL
							Evaluation

Figure 1.6: Example of claiming Objectives: Player sheet

■ **Example 1.3** In this example, the player satisfies the condition to claim Objective cards below. The reasons are as follow.

<b>S</b> $M_{3,3}$ 13	<b>C</b> $K(2,3)$ 11
This Mongolian tent graph is graceful. This means that we can label the vertices of this graph with $0, 1, \dots, 14$ in such a way that we can derived a labelling of edges that uses all the number from 1 to 14.	The $K$ in the name came from the king chess piece because this graph represent the legal move the piece can perform in an $m \times n$ chessboard.

Figure 1.7: Example of claiming Objectives: Objective to be claimed

**Definition 1.4.1 — Isomorphism.** Two graphs are isomorphic if we can morph one graph to the other without changing the configuration of its vertices and edges. We can see that in the example above the graph on the left is isomorphic to the graph in C Objective card.

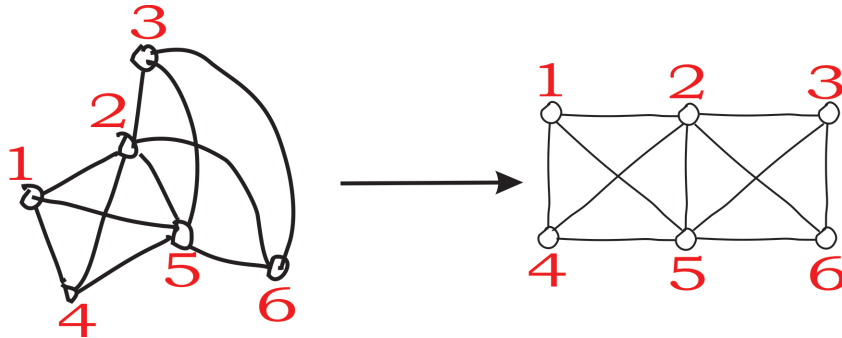


Figure 1.8: Transforming one graph to the other

**Definition 1.4.2 — Subgraph.** A subgraph is a graph contained in another graph, this includes all its vertices and all its edges. In this example, the graph on the bottom right contains the graph in S Objective card.

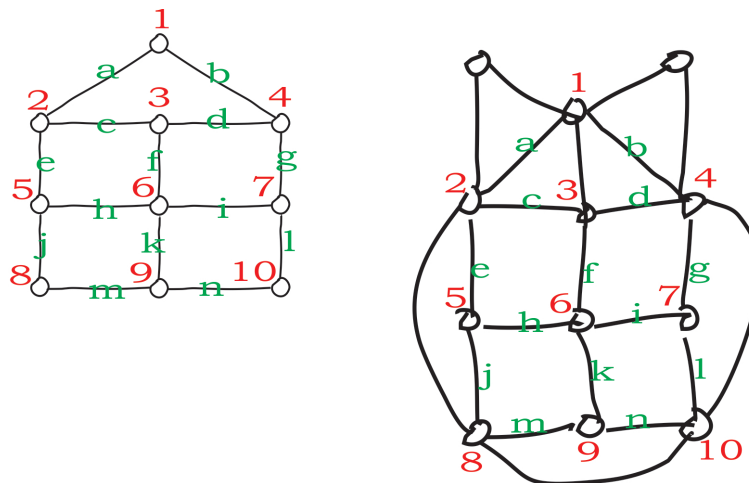


Figure 1.9: One graph is a subgraph of the other graph

**R** Labelling the vertices or the edges or both often helps us see more clearly isomorphism between graphs and whether one graph is a subgraph of another graph or not.

■

### 1.4.4 Bonus Cards

These cards will give us bonus points at the end of the game if we satisfy the requirement.

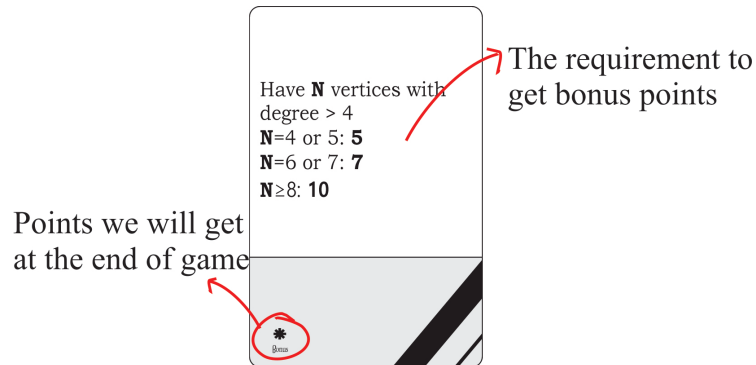


Figure 1.10: Bonus card anatomy

- Ⓡ There is one Bonus card that required us to construct a number of trees. In graph terms, a tree is a graph with no cycle subgraph.

### 1.4.5 Extra Power Cards

Extra Power cards have various effects that we can apply to our game. To have access to the effect of the current active Extra Power, we have to spend our currency, Eulerians. The sum that we need to spend is printed on the Extra Power card. There will be a new Extra Power card every round.

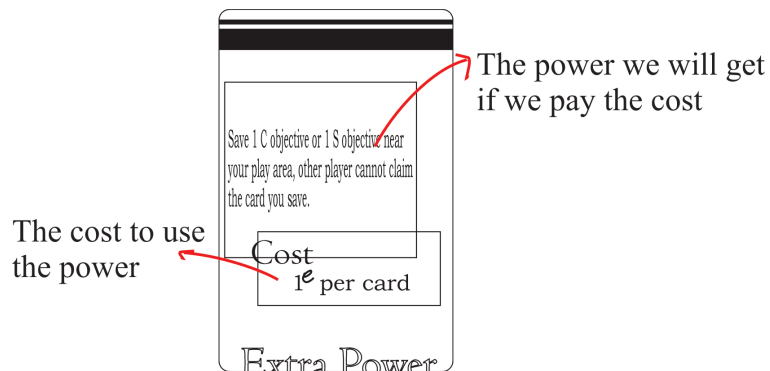


Figure 1.11: Extra Power card anatomy

## 1.5 Scoring

We will use the same player sheet in **Example 1.3** in to illustrate how we score in this game.

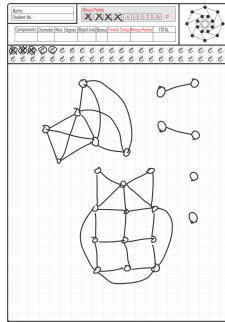


Figure 1.12: Player sheet example for scoring

### 1.5.1 Components of a Graph

In layman's terms, component of a graph is a subgraph that is not connected to other subgraph. We can see that the graph in the example above has 6 components. Below is the more rigorous definition.

**Definition 1.5.1** A subgraph of a graph which every two vertices of that subgraph are connected and no other vertex connected to that subgraph is called a component.

We will define the term "connected" in a more precise manner in **Definition 1.5.5**.

In this game, we classify two types of components.

1. Trivial component: consists solely of one vertex. These components contribute **negatively** to the score. Put how many trivial component one player have in "Trivial Comp."
2. Non-trivial component: as the name suggest, this component consists of more than one vertex. Put how many non-trivial component one player have in "Component".

So, we score **4 in "Components"** and **-2 in "Trivial Comp."** for our example.

### 1.5.2 Diameter of a Graph

Before we dive into the definition of diameter of a graph, we have to define a few things.

**Definition 1.5.2** A path between two vertices  $a$  and  $b$  is an alternating sequence between vertices and edges which start at  $a$  and finish at  $b$ . The length of a path is the number of edges in that path.

**Definition 1.5.3** Distance between two vertices  $a$  and  $b$  is the shortest path from  $a$  to  $b$ .

Here is the example of the concept above.

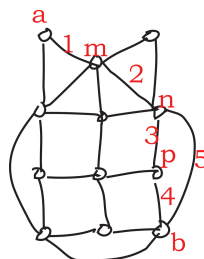


Figure 1.13: Path, length of path, and distance example



We can see that the sequence  $a, 1, m, 2, n, 3, p, 4, b$  is a path with length 4 while the distance between  $a$  and  $b$  is 3, with one of the path between  $a$  and  $b$  of length 3 is  $a, 1, m, 2, n, 5, b$ .

**Definition 1.5.4** Diameter of a graph is the greatest distance between any pair of vertices in that graph.

The diameter of the graph in our example is 3. So we put **3** in "**Diameter**".

**R** Determining the diameter of a graph is a bit trickier than other scoring components. So, please be patient when scoring this. It is easier to do this for each graph component first, then comparing the diameter between components. The highest one is the diameter of that graph.

We will go further into this concept. But before that, we need to define one more thing.

**Definition 1.5.5** Two vertices  $a$  and  $b$  are **connected** if there is a path that starts from  $a$  to  $b$  or the other way around.

If there is no such path, then  $a$  and  $b$  are **disconnected**.

Mathematically speaking, if two vertices are disconnected, their distance is infinite ( $\infty$ ). That is why in this game, we ignore all disconnected pair of vertices. This means that we do not need to consider the distance of any two vertices that come from different components, including trivial components.

### 1.5.3 Degree of a Vertex

We move on to the last concept in graph theory used in the scoring for this game.

**Definition 1.5.6** Degree of a vertex is the number of edges that come out of that vertex. Naturally, the maximum degree of a graph, usually noted as  $\Delta$ , is the maximum degree of its vertices.

In our example we have the maximum degree is 5. So we put **5** in "**Max. Degree**".

Let us say that the player in the example claim no other Objective cards besides these two:

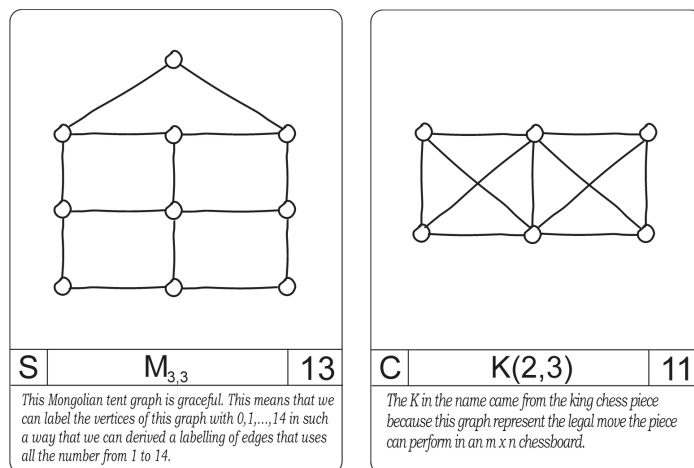


Figure 1.14: Example of claiming Objectives: Player sheet

and successfully finish the objective in the Bonus card:

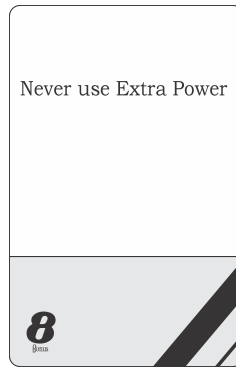


Figure 1.15: Example of claiming Objectives: Player sheet

This will give the player an additional **24 points in "Objective"** and **8 points in "Bonus"**. The last one to consider is Minus Points. We put the number on the left-most uncrossed box as our number. If all the boxes are crossed, then we put -17 in our Minus Points. For our example, we put **-6 in "Minus Points"**. So, our total score for this sheet is  $4 + 3 + 5 + 24 + 8 - 2 - 6 = 36$ . If this player has other sheets, then they also score those sheets using the same rules for each of their sheet. The total score is the sum of all their total score from every sheet. Let us say that for this example, this is the only sheet we played, then our total score is 36.

Components	Diameter	Max. Degree	Objectives	Bonus	Trivial Comp.	Minus Points	TOTAL
4	3	5	24	8	-2	-6	36

Figure 1.16: Final score




## 2. Single-Player and Multiplayer Mode

Let  $E$  be Edge,  $V$  be Vertex, and  $P$  be Number of Players.

### 2.1 Setup

1. Shuffle Extra Power cards deck, draw one and place them face-up. Place the rest of the deck face-down.
2. Shuffle  $S$  and  $C$  objective cards deck, draw five then place them face-up. Place the rest of the deck face-down.
3. Shuffle resource cards, draw two cards then place them face up. Place the rest of the deck face-down.
4. All players gain 2 Eulerians.
5. (Optional) Shuffle Bonus deck and dealt two cards to each player. Each player then discard one face-down and keep one near their playing area, keep them secret until the scoring phase.

 If you use Bonus deck, this will limit the player count to 2-10 players, as there are only 20 bonus cards. We can skip this step if we want to play with more than 10 players.

### 2.2 Gameplay

1. Every player choose one card from the 2 face-up resource cards. Player can choose the same card already chosen by other player.
2. Every player have to draw all vertices and edges according to their chosen card+modifier (this modifier tied to face-up Extra Power card), if you cannot draw all the edges you have to draw, cross the box on Minus Points for each edge you cannot draw starting from the left-most uncrossed box. Every player gain Eulerian according to their chosen card. During this phase, any player can use the face-up Extra Power card once (except stated otherwise by the card) by paying its price.
3. After every player gain their resources, each player can claim at most ONE objective card per round. As there is no turn order, the fastest player who claim the objective card is the

one who will probably get the card. Other players can ask for proof of the legitimacy of the claim. The player who claim the card has to prove them.

If the player cannot prove the claim or the proof is wrong then they return the card and cross the left-most uncrossed box on their Minus Points. If the claim is proven then the player take the claimed objective card and store them near their player area. Draw a new objective card to replace it.

If no player contested the claim, then that player automatically get the card. Also draw a new objective card to replace it.

4. After all the player have their chance to claim objective cards, every player has the chance to discard all the objective cards and replace them with five new objective cards drawn from the deck, this action is called refresh. The player who refresh the objective cards must cross two boxes from their Minus Points, starting from the left-most uncrossed box.
5. Discard the face-up Extra Power card and draw a new one to replace it.
6. Discard all the face-up resource cards then draw two new cards and place them face up.

If during any of the phase, there exist a player who cannot cross the box from their Minus Points, then that player cannot draw anything on their sheet and must take a new sheet with the first two left-most boxes on their Minus Points crossed. This player can resume the game on their new sheet. The player with multiple sheets will score from all their sheet, including all their Minus Points.

If, at anytime any player has to draw a new Objective card and the Objective cards deck is empty, shuffle all discarded Objective cards pile to create a new Objective cards deck. Draw new Objective cards from this pile.

### 2.3 Endgame and Last Round

The game ends on the round when resource cards deck ran out. This last round proceeds until Step 3 of the gameplay (**Subsection 2.2**). If the Bonus cards deck is used in this game, all players reveal their Bonus card so the other players can see. Count the final score according to the rule in **Subsection 1.5**. The player with the highest score win the game. The tie-breaker is the following:

1. The player with the least crossed Minus Points boxes.
2. The player with the least trivial components.
3. The player with the highest maximum degree.
4. The player with the highest diameter.
5. The player with the most components.

If there is still a tie, then they share the same position.

## HOW THE ROUND PLAY:

1. Choose one from 2 face-up resource cards
  2. Gain vertices and edges and draw them, gain Eulerian, spend Eulerian on Extra Power
  3. Claim at most ONE objective card
  4. Resolve any dispute if any
  5. Refresh Objective card if any player desires to
  6. Discard Extra Power card and draw a new one
  7. Discard resource cards then draw two new cards
- The game ends when the resource cards deck is depleted. Proceed to Scoring phase.

## MINUS POINTS

Cross the left-most uncrossed box if you

1. cannot draw an edge
2. cannot prove your claim
3. do refresh action, cross two boxes

If you cross all 8 boxes and you have to cross another box,

1. store your sheet, do not draw anything there until the end of game
2. gain a new sheet, cross the first two left-most boxes
3. resume play in the new sheet

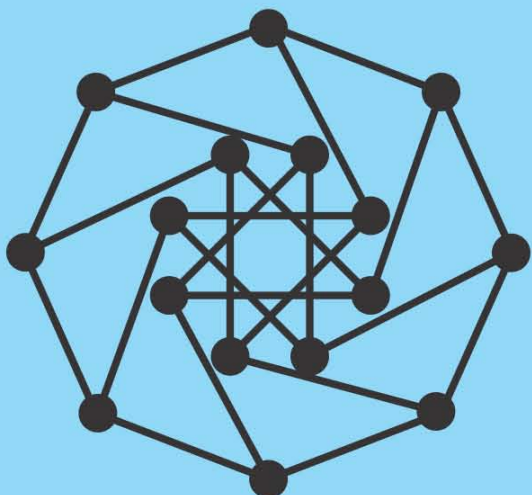
## SCORING

Each player do the following:

1. COMPONENTS: Count the number of non-trivial components
2. DIAMETER: Determine the greatest distance between any pair of vertices
3. MAX. DEGREE: Determine the maximum of all degree of vertices.
4. OBJECTIVES: Count all scores from claimed Objective Cards
5. BONUS: Put the score if you succeed in fulfilling the condition
6. TRIVIAL COMP.: Count the number of trivial components times -1
7. MINUS POINTS: Put the number in the left-most uncrossed box, or -17
8. Add the number in Step 1-7
9. Do Step 1-8 for your other sheets then add them together

## TIE-BREAKER

1. The least crossed Minus Points boxes.
2. The least trivial components.
3. The highest maximum degree.
4. The highest diameter.
5. The most components.



# Introduction to Graph Theory

*drawing*



Never use Extra Power

8  
Brim

Have no component with more than 12 edges

5  
Brim

Component with 2nd most number of vertices has  $N$  vertices  
 $N=5$  or 6: **5**  
 $N=7$  or 8: **8**  
 $N \geq 9$ : **10**

\*  
Brim

For every player that have at least  $2C$  at the end of the game, gain **3**

\*  
Brim

Have  $N$  trees as components  
 $N=3$  or 4: **3**  
 $N=5$  or 6: **5**  
 $N \geq 7$ : **8**

\*  
Brim

For every player that claimed at least 1  $C$  objective and at least 2  $S$  objectives, gain **3**

\*  
Brim

All vertices have the same degree at the end of the game

8  
Brim

Have only 1 component

7  
Brim

All vertices has degree at least 2 at the end of the game

7  
Brim

Have 1 vertex adjacent to all other vertices

8  
Brim

No vertex with degree  $> 3$

7  
Brim

Have  $N$  vertices with degree  $> 4$   
 $N=4$  or 5: **5**  
 $N=6$  or 7: **7**  
 $N \geq 8$ : **10**

\*  
Brim

Draw the longest path between all players

5  
Brim

Have at least  $6C$  at the end of the game

5  
Brim

Claimed only  $S$  objective cards

7  
Brim

Draw the longest cycle between all players

7  
Brim

For every player that have at least 3 components, gain **3**

\*  
Brim

Save 1 C objective near your play area, other player cannot claim the card you save. You can do this up to two times this round.

Cost  
1<sup>e</sup> per card

Extra Power

Add/ subtract 1 edge to what you have to draw this round.

Cost  
2<sup>e</sup>

Extra Power

Add/ subtract up to 2 edges to what you have to draw this round.

Cost  
3<sup>e</sup>

Extra Power

Add/ subtract up to 2 edges to what you have to draw this round.

Cost  
3<sup>e</sup>

Extra Power

Add/ subtract 1 vertex and/ or 1 edge to what you have to draw this round.

Cost  
2<sup>e</sup>

Extra Power

Add/ subtract up to 2 vertices and/ or 2 edges to what you have to draw this round.

Cost  
4<sup>e</sup>

Extra Power

Change 1 vertex to 1 edge from what you have to draw (and the other way around).

Cost  
2<sup>e</sup>

Extra Power

Change 1 vertex to 1 edge from what you have to draw (and the other way around). You can do this up to two times this round.

Cost  
2<sup>e</sup> per change

Extra Power

Save 1 C objective near your play area, other player cannot claim the card you save. You can do this up to two times this round.

Cost  
1<sup>e</sup> per card

Extra Power

Add/ subtract up to 2 vertices to what you have to draw this round.

Cost  
2<sup>e</sup>

Extra Power

Add/ subtract up to 2 vertices to what you have to draw this round.

Cost  
2<sup>e</sup>

Extra Power

Add/ subtract 1 vertex to what you have to draw this round.

Cost  
1<sup>e</sup>

Extra Power

Add/ subtract 1 vertex to what you have to draw this round.

Cost  
1<sup>e</sup>

Extra Power

Add/ subtract 1 edge to what you have to draw this round.

Cost  
2<sup>e</sup>

Extra Power

Save 1 S objective near your play area, other player cannot claim the card you save. You can do this up to two times this round.

Cost  
1<sup>e</sup> per card

Extra Power

Save 1 C objective or 1 S objective near your play area, other player cannot claim the card you save.

Cost  
1<sup>e</sup> per card

Extra Power

Draw 3 Bonus cards, discard 1 Bonus card.

Cost  
3<sup>e</sup>

Extra Power

Draw 2 Bonus cards, discard 1 Bonus card.

Cost  
3<sup>e</sup>

Extra Power

Save 1 S objective near your play area, other player cannot claim the card you save. You can do this up to two times this round.

Cost  
1<sup>e</sup> per card

Extra Power

Save 1 C objective or 1 S objective near your play area, other player cannot claim the card you save.

Cost  
1<sup>e</sup> per card

Extra Power

Draw 3 Bonus cards, discard 1 Bonus card.

Cost  
3<sup>e</sup>

Extra Power

Draw 2 Bonus cards, discard 1 Bonus card.

Cost  
3<sup>e</sup>

Extra Power

Add/ subtract 1 edge to what you have to draw this round.

Cost  
2<sup>e</sup>

Extra Power

Add/ subtract 1 vertex to what you have to draw this round.

Cost  
1<sup>e</sup>

Extra Power

Add/ subtract 1 vertex to what you have to draw this round.

Cost  
1<sup>e</sup>

Extra Power

Add/ subtract up to 2 vertices to what you have to draw this round.

Cost  
2<sup>e</sup>

Extra Power

Add/ subtract up to 2 vertices to what you have to draw this round.

Cost  
2<sup>e</sup>

Extra Power

Save 1 C objective near your play area, other player cannot claim the card you save. You can do this up to two times this round.

Cost  
1<sup>e</sup> per card

Extra Power

Add/ subtract up to 2 vertices to what you have to draw this round.

Cost  
2<sup>e</sup>

Extra Power

Add/ subtract up to 2 vertices to what you have to draw this round.

Cost  
2<sup>e</sup>

Extra Power

Add/ subtract 1 vertex to what you have to draw this round.

Cost  
1<sup>e</sup>

Extra Power

Add/ subtract 1 vertex to what you have to draw this round.

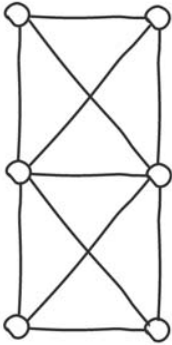
Cost  
1<sup>e</sup>

Extra Power

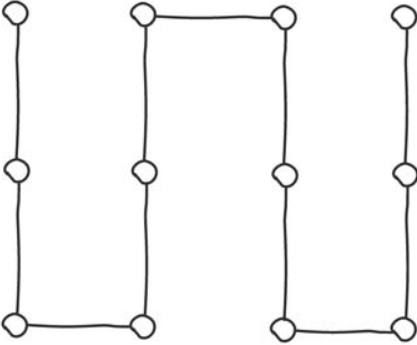
Add/ subtract 1 edge to what you have to draw this round.

Cost  
2<sup>e</sup>

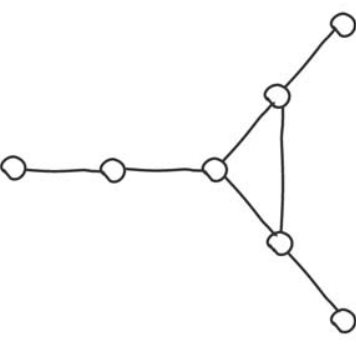
Extra Power



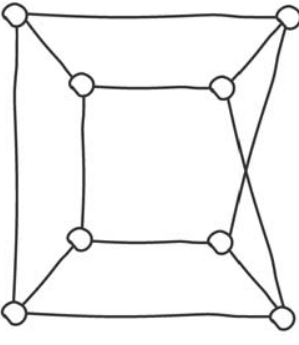
<b>C</b>	<b>K(2,3)</b>	<b>11</b>
<p>The <i>K</i> in the name came from the king chess piece because this graph represent the legal move the piece can perform in an <math>m \times n</math> chessboard.</p>		



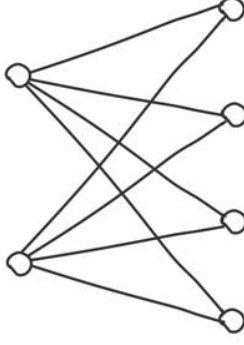
<b>C</b>	<b>P<sub>12</sub></b>	<b>17</b>
<p>This graph is a part of path graphs family. Except for one graph, there is no graph that does not contain path graph(s) as its subgraph(s).</p>		



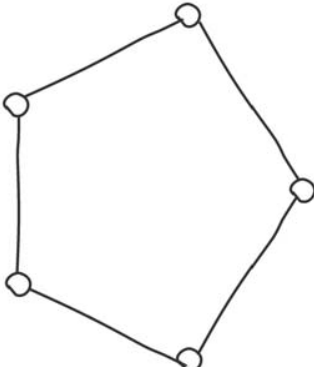
<b>C</b>	<b>(3,2)-fan</b>	<b>10</b>
<p>Other name this graph is Eiffel Tower graph due to its similarity to the tower and also because it is the most famous.</p>		



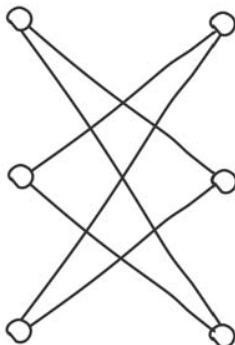
<b>C</b>	<b>M<sub>4</sub></b>	<b>14</b>
<p>It is also called Wagner graph. It is a part of Möbius ladders family. It is obtained by cross-connecting two ends of a ladder graph. If you are familiar with Möbius strip, it is also clear why it is named so.</p>		



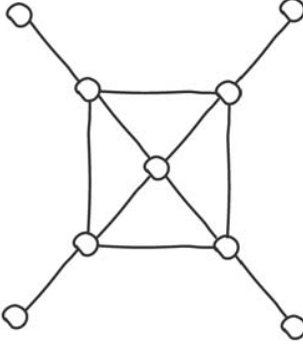
<b>C</b>	<b>K<sub>2,4</sub></b>	<b>10</b>
<p>This graph is an example of a complete bipartite graph. It is complete bipartite because the vertices are grouped into two partitions such that there is no vertex adjacent to other vertex in the same partition and each vertex adjacent to every vertex in the other partition.</p>		



<b>C</b>	<b>C<sub>5</sub></b>	<b>7</b>
<p>The cycle graphs, as the name suggest, consist only one cycle that goes through all vertices. This graph has 5 vertices, that is why its name is C<sub>5</sub>.</p>		



<b>C</b>	<b>P<sub>2</sub> x K<sub>3</sub></b>	<b>9</b>
<p>Another name for this graph is 3-crown graph. We can also get this graph from a complete bipartite graph by removing all vertical edges</p>		



<b>C</b>	<b>5-Helm</b>	<b>15</b>
<p>This graph is constructed from 5-wheel graph or W<sub>5</sub> by attaching one edge and one vertex to every outer vertex of the wheel. This is called 5-helm graph, or H<sub>5</sub> for short.</p>		



<b>C</b>	<b>4-Crossed Prism</b>	<b>14</b>
<p>We can see that this graph is a prism with crossed edges. Due to how the edges cross, the family of <math>n</math>-crossed prism graph can only be defined if <math>n</math> is even.</p>		

<b>C</b>	<b><math>K_3 \cup K_4</math></b>	<b>11</b>
<p>At a glance this graph can be seen as two graphs drawn close to each other. But because we name them <math>K_3 \cup K_4</math> it counts as one graph... with two components.</p>		

<b>C</b>	<b><math>3K_3</math></b>	<b>11</b>
<p>The graph <math>K_3</math> is the only complete graph that is also a cycle.</p>		

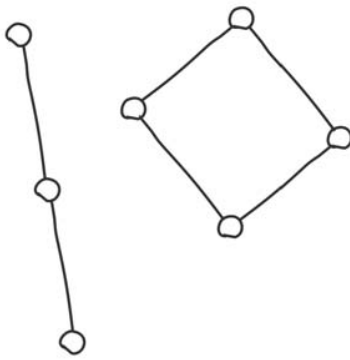
<b>C</b>	<b>6-pan</b>	<b>10</b>
<p>Another name for this graph is <math>(6, 1)</math>-tadpole graph. In fact, any <math>n</math>-pan graph is an <math>(n, 1)</math>-tadpole graph.</p>		

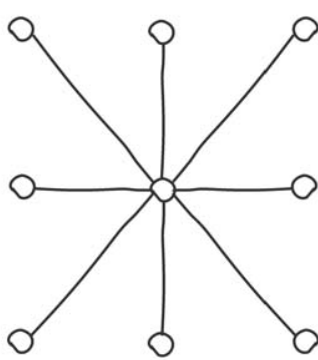
<b>C</b>	<b><math>P_3 \square P_3</math></b>	<b>15</b>
<p>An easier example on how to do a Cartesian product between two graphs is by producing two path graphs, like this graph. This example also give some justifications on the product name.</p>		

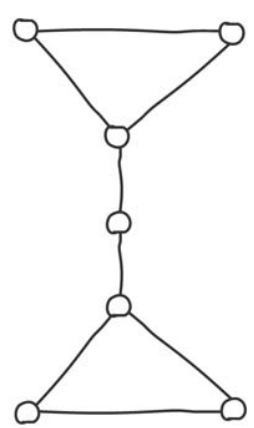
<b>C</b>	<b>3-Barbell</b>	<b>9</b>
<p>This graph is constructed from 2 complete graphs <math>K_3</math> connected with a bridge. Other barbell graphs also constructed with similar manner but with other complete graphs.</p>		

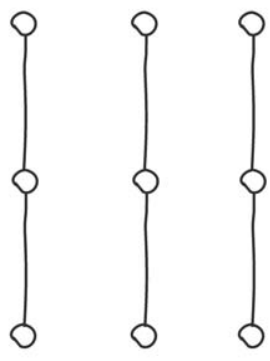
<b>C</b>	<b>4-Prism</b>	<b>14</b>
<p>This graph represents a prism with rectangular base. The 4 in the name came from the rectangle, or <math>C_4</math> in this context.</p>		

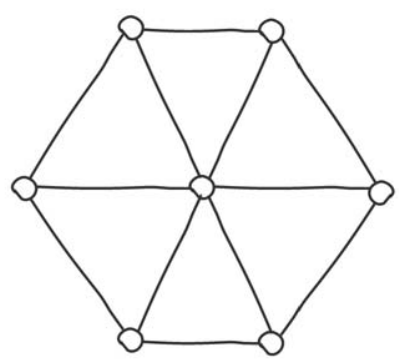
<b>C</b>	<b><math>P_3 \times P_3</math></b>	<b>13</b>
<p>The operator <math>X</math> is called Kronecker or tensor or categorical product. This graph in particular is also <math>S_4 \cup C_4</math>.</p>		

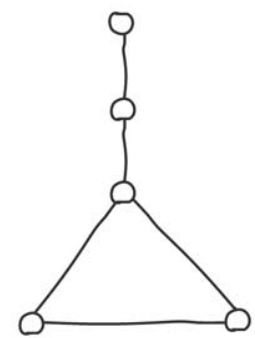
	<b>C</b> $P_3 \cup C_4$	<b>10</b>
<p>The union operator <math>\cup</math> of some graphs is the easier way to construct a new graph from the graphs we already have.</p>		

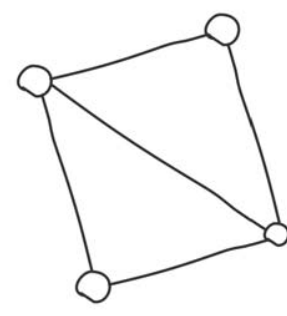
	<b>C</b> $S_8$	<b>13</b>
<p>In graph theory, a star <math>S_n</math> is a tree with one internal vertex and <math>n</math> leaves (a leaf in a tree is a vertex with exactly one edge attached).</p>		

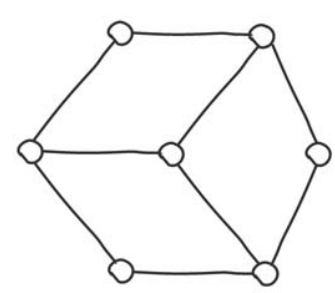
	<b>C</b> $KP(3,3,2)$	<b>11</b>
<p>This is an example of kayak paddle graph. This family of graphs is formed by joining a cycle <math>C_m</math> and <math>C_n</math> by a path <math>P_k</math> and we call them <math>KP(m,n,k)</math>.</p>		

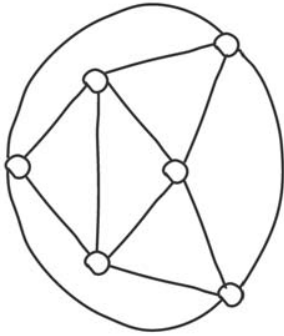
	<b>C</b> $3P_3$	<b>11</b>
<p>Path graphs always have one less edge than its vertices while cycle graphs have equal numbers of edges and vertices.</p>		

	<b>C</b> $W_7$	<b>13</b>
<p>In graph theory, a wheel is a star with a cycle added to the outer vertices.</p>		

	<b>C</b> $T_{3,2}$	<b>7</b>
<p>This is a tadpole graph, or a dragon graph, or a kite graph. Anything goes really as long as it makes sense.</p>		

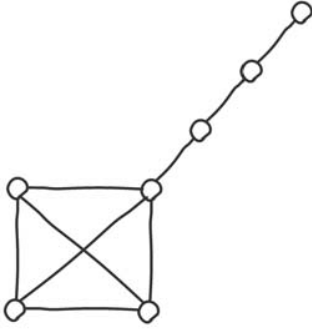
	<b>C</b> <b>Diamond</b>	<b>6</b>
<p>We can construct this graph from <math>K_4</math> by removing exactly one edge, any edge.</p>		

	<b>C</b> $G_3$	<b>11</b>
<p>This gear graph is constructed from a wheel graph by adding a vertex into each pair of adjacent outer vertices.</p>		



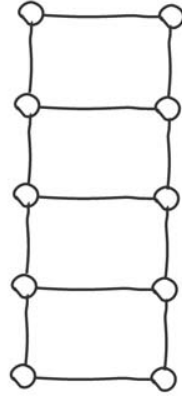
**C** 3-antiprism **12**

*This graph represent a polyhedron formed by 8 triangles. Two of the triangles act as the top and bottom while the other six are the wall.*



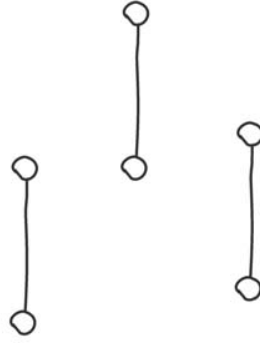
**C**  $L_{4,3}$  **11**

*This graph is a part of lollipop graphs family because it consists of a complete graph and a path graph connected by a bridge. Of course not, it is called lollipop graph because it looks like a lollipop.*



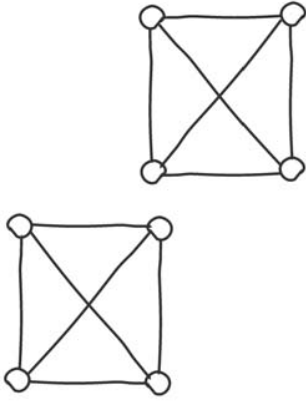
**C**  $L_5$  **16**

*The L in the name came from ladder. It is unclear why though.*



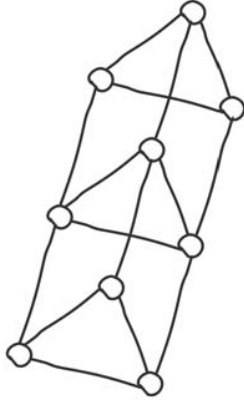
**C**  $3P_2$  **7**

*The name  $3P_2$  is an abbreviation of  $P_2 \cup P_2 \cup P_2$ . The graph  $P_2$  is also a complete graph with the smallest number of vertices.*



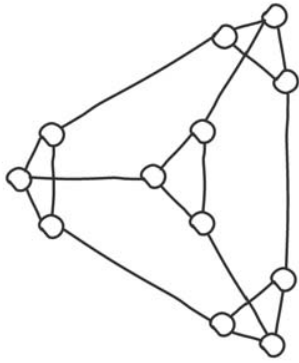
**C**  $2K_4$  **14**

*We can also call this graph as  $K_4 \cup K_4$ , but we usually write it  $2K_4$  to simplify it.*

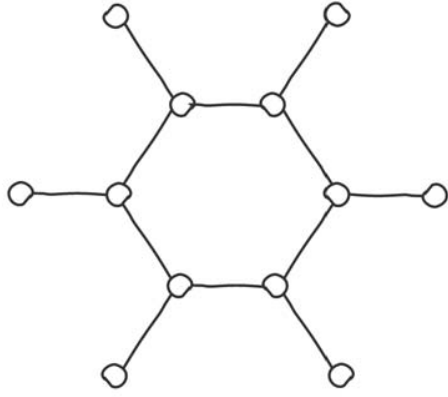


**C**  $K_3 \square P_3$  **16**

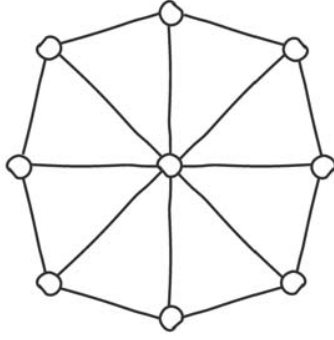
*This graph is the result of Cartesian product between  $K_3$  and  $P_3$ .*



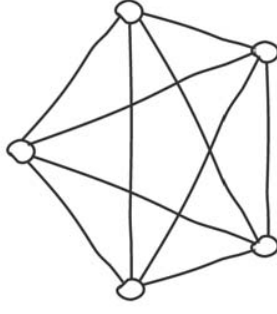
**S** Truncated Tetrahedron **12**  
*This graph is a skeleton of a solid constructed from a tetrahedron with each of its corner is truncated.*



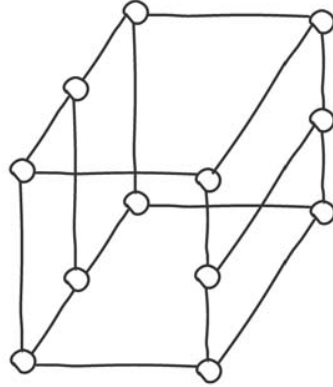
**S**  $C_6 \odot K_1$  **14**  
*This graph product is called corona product. In this case, we take one copy of  $C_6$  and 6 copy of  $K_1$ , then we connect each vertex in  $C_6$  to a different copy of  $K_1$ .*



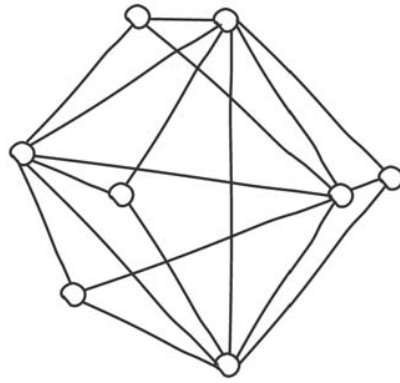
**S**  $W_9$  **12**  
*If we remove all edges on the outer cycle, we will get a star, in this case  $S_9$ . This does not mean it also lose a vertex, it is just the naming convention we use.*



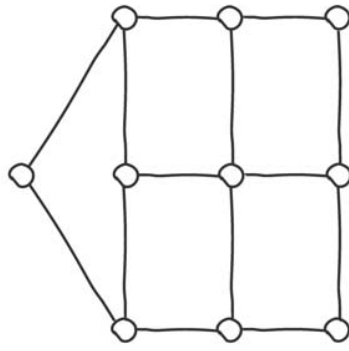
**S**  $K_5$  **12**  
*A graph is complete if every vertex is adjacent to every other vertex. We can also see it like this: a graph is complete if we cannot add any more edges to that graph.*



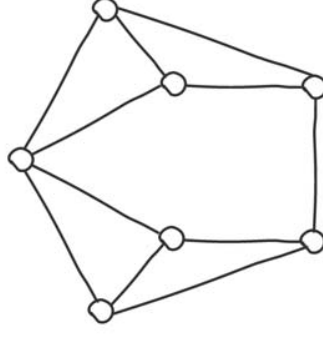
**S** Bidiakis Cube **16**  
*In this graph we can find a cycle (a path with the same beginning and end) that goes through every vertex exactly once. This kind of graph is called Hamiltonian graph, named after William Rowan Hamilton.*



**S** Triakis Tetrahedron **11**  
*This graph is a skeleton of triakis tetrahedron, a solid constructed from a tetrahedron with a triangular pyramid added to each of its face.*



**S**  $M_{3,3}$  **13**  
*This Mongolian tent graph is graceful. This means that we can label the vertices of this graph with  $0, 1, \dots, 14$  in such a way that we can derive a labelling of edges that uses all the number from 1 to 14.*



**S** Moser Spindle **9**  
*This graph is also known as Hajós graph. It is also a unit-distance graph (a graph that can be drawn with edges of the same lengths).*

<b>S</b>	<b>Krachkardt Kite</b>	<b>13</b>
<p>This graph is named after David Krachkardt who first use this graph in social network theory.</p>		

<b>S</b>	<b>&lt;unnamed&gt;</b>	<b>9</b>
<p>This graph is constructed by Oliver Schaudt and Vera Weil in 2015 as one of many examples of graphs with certain conditions.</p>		

<b>S</b>	<b>Franklin Graph</b>	<b>16</b>
<p>Let all vertices in a graph have the same degree (the number of edges attached to them). The graphs with this property are called <i>n</i>-regular graphs, with <i>n</i> is the degree of any vertex in the graph. In this case, Franklin graph is a 3-regular graph.</p>		

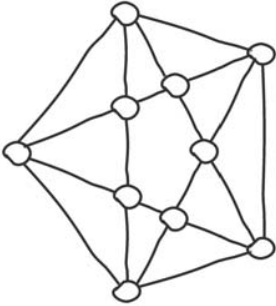
<b>S</b>	<b><math>\overline{P_3UP_3}</math></b>	<b>8</b>
<p>This graph is obtained from two identical path graphs and using complement and union operators. We can see that the result does not resembles path graph at all.</p>		

<b>S</b>	<b>Bicorn</b>	<b>10</b>
<p>This graph and its cousin, Tricorn, played important roles in solving several problems in graph theory, one in particular is Lovász conjecture.</p>		

<b>S</b>	<b>Golomb Graph</b>	<b>14</b>
<p>This graph is an example of unit-distance graph, a graph which can be drawn with all edges has the same length. Obviously, the graph above is not a unit-distance representation.</p>		

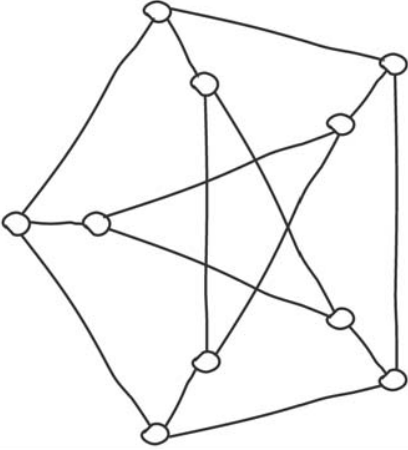
<b>S</b>	<b>Co-Eiffel Tower</b>	<b>10</b>
<p>A graph containing Co... in its name usually means it is a complement to some graph. In this case, it is the complement of Eiffel Tower graph.</p>		

<b>S</b>	<b>K(2,4)</b>	<b>11</b>
<p>The graphs family this graph belongs to is not the only family of graphs based on the movement of a chess piece. There are also rook's graph and knight's graph (or knight's tour graph).</p>		



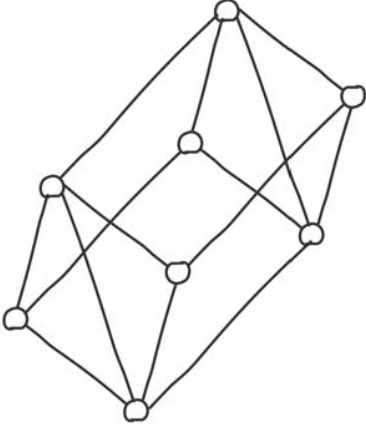
**S** 5-Antiprism 14

At a glance, this graph is very similar to  $K_5$ , but this graph has 5 more vertices and 15 more edges.



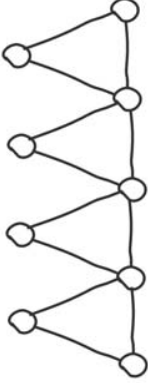
**S** Petersen Graph 13

This graph is named after Julius Petersen who constructed it. This graph is also used in many problems in graph theory as example or counterexample.



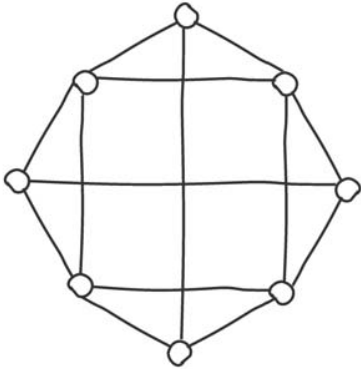
**S** Diamond  $\square P_2$  11

There is no edge in diamond graph that if we remove them, the resulted graph is a path.



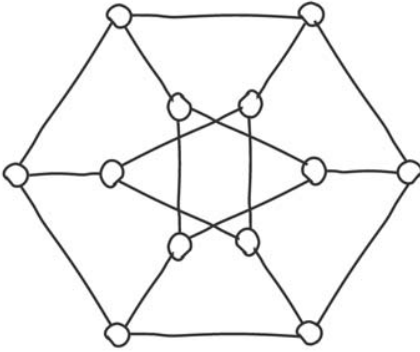
**S**  $TS_9$  11

$TS_9$  is an example of a matchstick graph. As the name suggest, a matchstick graph is a graph with all its edges have the same length and it is a planar graph, a graph with no edge overlap each other.



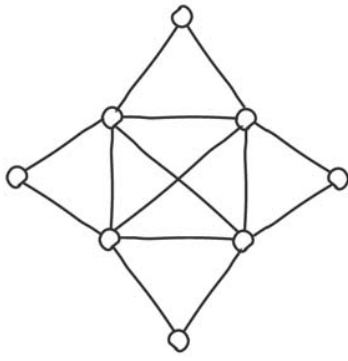
**S** 9-Paley 12

The way this graph and all other  $n$ -Paley graphs constructed is closely related to other field in mathematics, algebra, or to be more specific, finite fields.



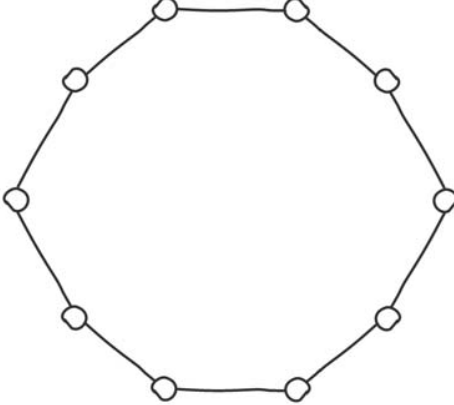
**S** Dürer Graph 16

This graph is named after Albrecht Dürer who painted Melencolia I that contains this graph as a solid, now also named after him, Dürer solid.



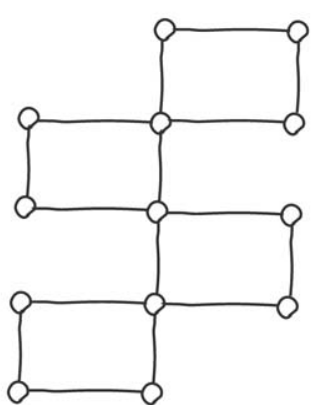
**S**  $S(4)$  11

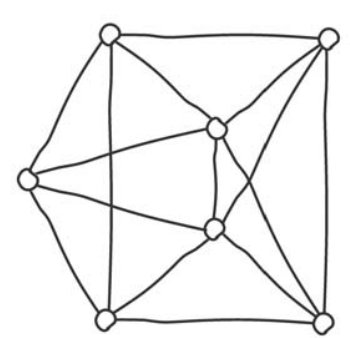
$a$

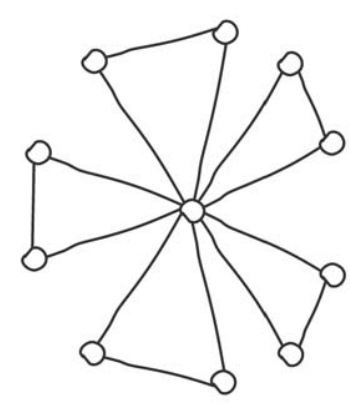


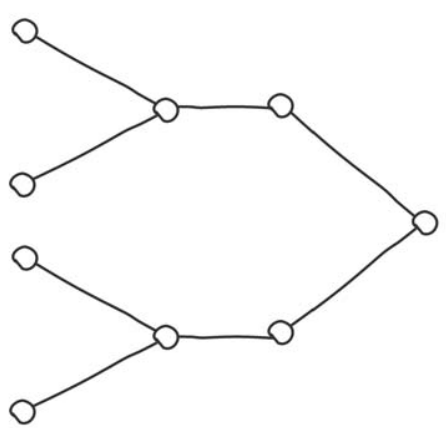
**S**  $C_{10}$  11

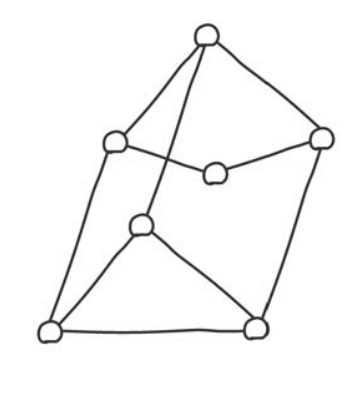
All cycle graphs are not only Hamiltonian, but they are also Eulerian (a graph that contains a cycle that goes through all its edges exactly once).

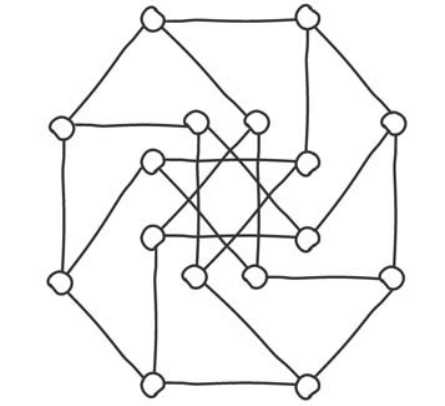
	<b>S</b> Twindragon Level 2	<b>16</b>
<i>It is self-explanatory on why the name of this graph is twindragon level 2.</i>		

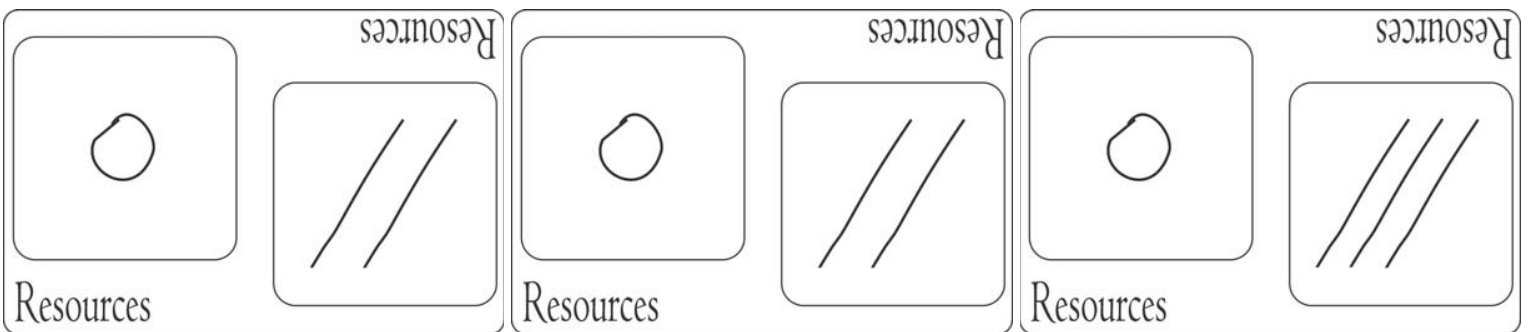
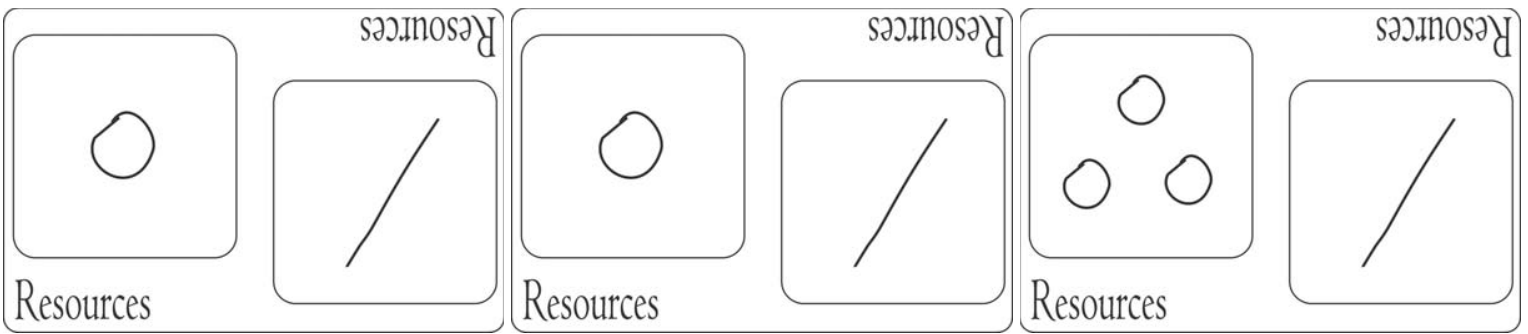
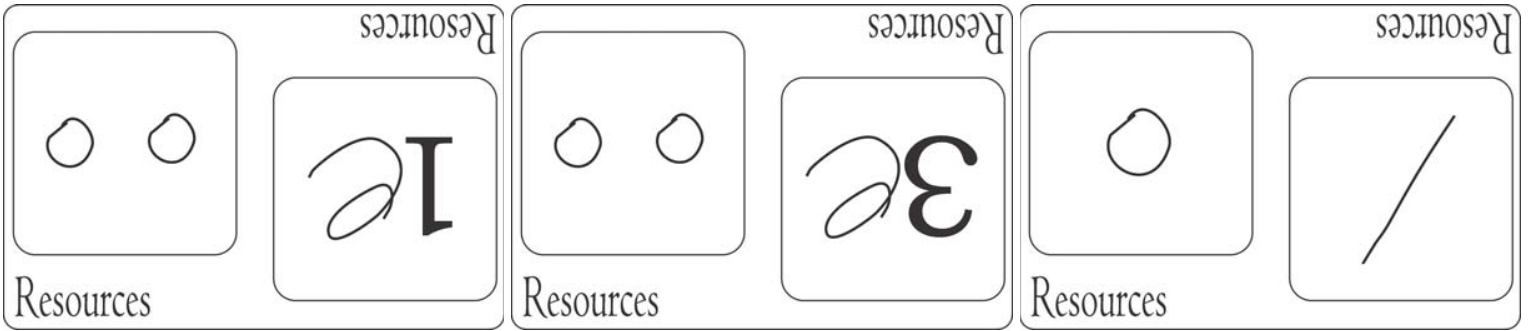
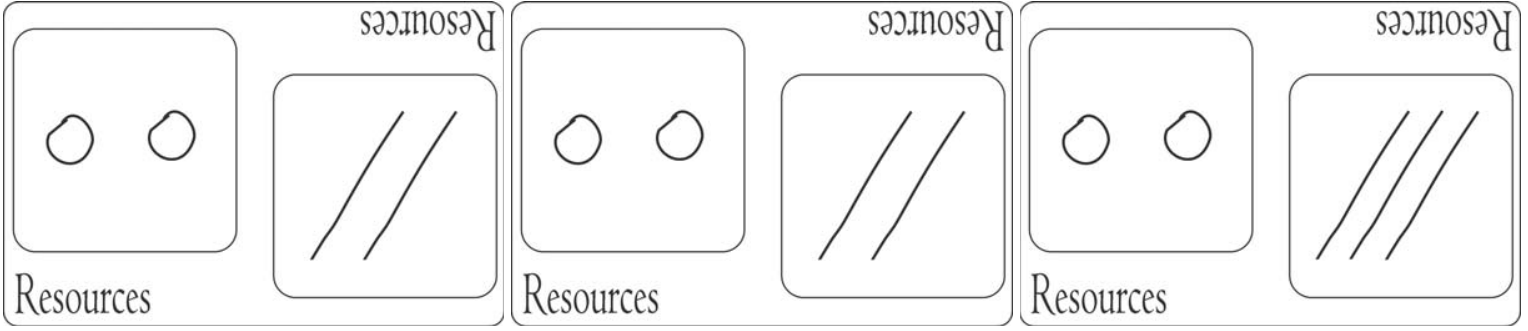
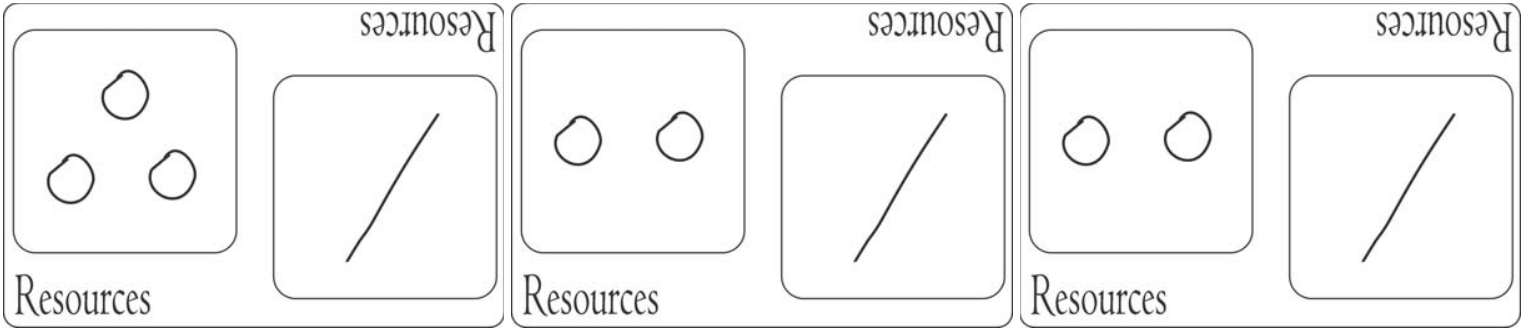
	<b>S</b> $\overline{P}_7$	<b>10</b>
<i>The overline in the name of this graph means complement in the sense of adjacency. This means that two vertices is adjacent in the complement graph if and only if they are not adjacent in the original graph.</i>		

	<b>S</b> $F_5$	<b>14</b>
<i>This graph has the property: any two vertices has exactly one common neighbor (other vertex adjacent to both vertices).</i>		

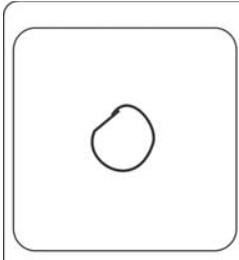
	<b>S</b> $B_{2,4}$	<b>10</b>
<i>This is one of banana tree graphs. Tree in graph theory is defined as a graph that contain no cycle, so it makes sense that it is a tree. As for the banana part, we are not quite sure.</i>		

	<b>S</b> $H_{3,7}$	<b>8</b>
<i>c</i>		

	<b>S</b> Möbius-Kantor	<b>21</b>
<i>August Ferdinand Möbius asked for the existence of polygons that satisfies some configurations and some decades later Seligmann Kantor provided them. This graph related to that configuration. Hence the name.</i>		



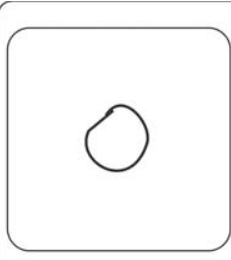




Resources



Resources



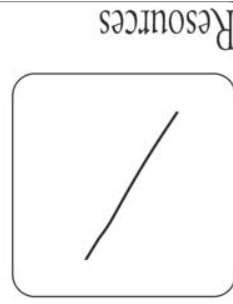
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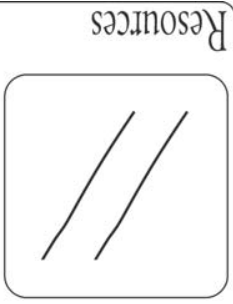
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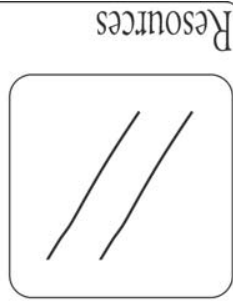
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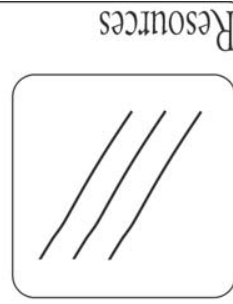
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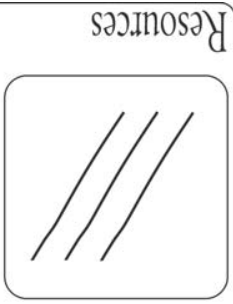
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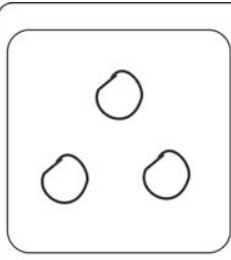
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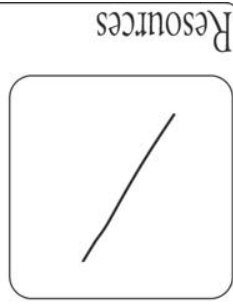
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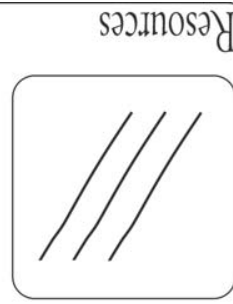
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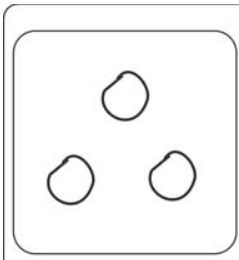
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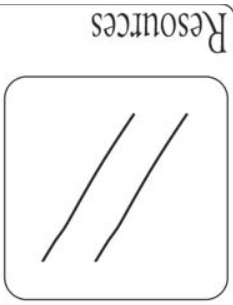
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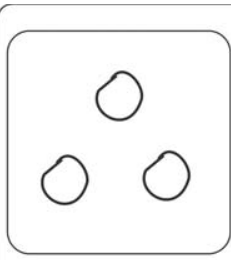
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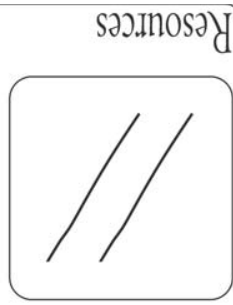
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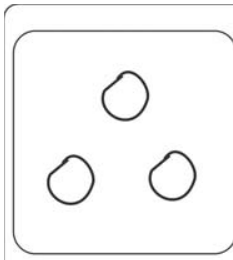
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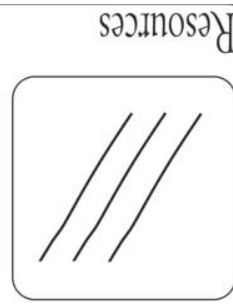
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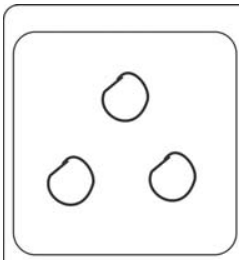
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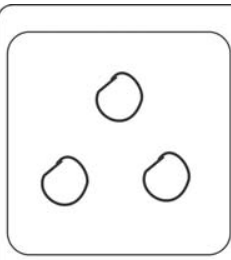
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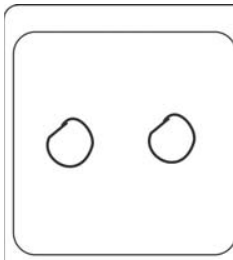
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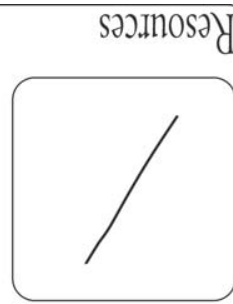
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Resources



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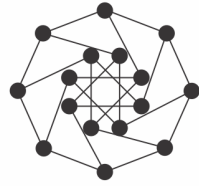


Resources

Name : \_\_\_\_\_  
Student No. : \_\_\_\_\_

Minus Points

0 -1 -2 -4 -6 -8 -11 -14 -17



Components	Diameter	Max. Degree	Objectives	Bonus	Trivial Comp.	Minus Points	TOTAL

